POSSIBLE LIMITS ON PHOTON PROPAGATION
FROM QUANTUM GRAVITY AND SPACE-TIME FOAM

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ABSTRACT

Many quantum gravity theories imply that the vacuum is filled with virtual black holes. This paper explores the process in which high energy photons interact with virtual black holes and decay into gravitons and photons of lower energy. The effect requires violation (or modification) of Lorentz invariance and implies that high energy photons cannot propagate over arbitrarily large distances. For the standard Planck mass and the likely form for the interaction cross section, this quantum foam limit becomes $d_* < 450$ Mpc $(E_\gamma/10^7 \text{GeV})^{-5}$. For quantum gravity theories that posit a lower Planck scale, the interaction rate is larger and the limit is stronger. This paper uses extant observations of gamma rays from cosmological sources to constrain this process for varying values of the Planck mass and a range of forms for the interaction cross sections.

Quantum gravity currently lacks definitive experimental tests. However, most theories of quantum gravity predict that space is filled with virtual black holes, which can absorb photons and re-radiate them as the black holes evaporate. If energy and momentum are conserved during such an interaction, the virtual black hole will usually re-radiate a photon with the same energy and direction as the original (absorbed) photon. In general, the phase of the emitted photon will be different from that of the absorbed photon and the photon is delayed by a small time interval $\Delta t \sim M_{\text{pl}}^{-1}$, but these processes do not affect most observations. In addition to phase changes and time delays, however, virtual black holes can also radiate multiple particles, provided that the relativistic dispersion relation has a modified form (as predicted by many versions of quantum gravity). In spite of its lower probability, this latter effect is more readily observable and can be used to constrain theories of quantum gravity. This paper uses existing observations of high energy photons to place constraints on this process.

For astronomical sources that are close enough so that cosmic expansion can be neglected, the optical depth τ for photons to interact with virtual black holes takes the simple form

$$\tau = n\sigma d_* , \tag{1}$$

where n is the number density of virtual black holes, σ is the interaction cross section, and d_* is the distance from the astronomical source. (This expression is generalized below to include cosmic expansion). A successful observation of an astronomical source implies that $\tau \leq 1$.

According to the scenario of virtual black holes filling the vacuum – the space-time foam – the vacuum contains about one Planck mass black hole per Planck volume [1–3]. The number density of virtual black holes thus takes the form

$$n = \alpha M_{\rm pl}^{3} , \tag{2}$$

where $M_{\rm pl}$ is the Planck mass and α is a dimensionless constant of order unity. [We use units in which $\hbar = 1$, $c = 1$, $G = M_{\rm pl}^{-2}$, and the Planck length $\ell_{\rm pl} = M_{\rm pl}^{-1}$.]

Next we need to specify the cross sections for photons interacting with virtual black holes. The geometrical cross section is $\sigma_0 \approx \pi \ell_{\rm pl}^{2}$. Since all photons of astronomical interest are in the long wavelength regime $\lambda \gg \ell_{\rm pl} = M_{\rm pl}^{-1}$, the absorption cross section σ_1 is highly suppressed relative to σ_0. Large black holes are thought to emit radiation with a nearly thermal spectrum [4]. If perturbed virtual black holes act similarly and the absorption cross section is the same as the emission cross section, the long wavelength limit of the absorption cross section takes the form $\sigma_1 = \beta_1 \pi \ell_{\rm pl}^{2} (\ell_{\rm pl}/\lambda)^2$ $= \beta_1 \pi E_\gamma^{2} M_{\rm pl}^{-4}$, where β_1 is a dimensionless parameter and the energy $E_\gamma = \lambda^{-1}$. For classical photon fields interacting with a static, uncharged, non-rotating (Schwarzschild) black hole, the absorption cross section has been calculated [5]; in the long wavelength limit, one finds $\beta_1 = 64/3$ (although quantum effects could modify this value). This absorption cross section implies an optical depth $\tau_1 = \alpha \beta_1 \pi d_1 \ell_{\rm pl} \lambda^{-2}$. The path length d_1 required for $\tau_1 > 1$ takes the form $d_1 \approx 1$ Mpc $(\lambda/1\mu{\rm m})^2$ and is thus astronomically interesting for optical photons. In most cases, however, the virtual black hole will emit a photon with the same energy (but with a different phase) and the absorption event would be impossible to detect.

Here we consider the case where the absorption of a photon by a virtual black hole leads to the emission of two particles rather than one. In order to conserve energy and momentum, the outgoing

particles must travel parallel to the incoming photon. When a photon (spin-1) emerges, the second particle must be a graviton (spin-2) to conserve spin angular momentum. In conventional particle physics, this process, sometimes called photon splitting, is not generally allowed for two reasons: (a) The phase space for the outgoing particles vanishes because the momenta are all parallel, and (b) The amplitude vanishes because the contractions of the momenta with each other (or with the polarizations) vanish [6]. However, the process is allowed if it violates (or modifies) Lorentz symmetry. For example, the dispersion relation for massless particles could have an additional term [7], $E_\gamma{}^2(p) = p^2 + \xi p^{n+2}/M_{\rm pl}{}^n$. In order for the phase space to have non-vanishing volume, at least one of the outgoing particles must have a Lorentz-violating (or modifying [7]) factor $f \sim \xi(p/M_{\rm pl})^n$; this same factor allows the matrix elements to be nonvanishing as well. As a result, we expect the cross section σ_2 for photon absorption and re-radiation of two particles to take the general form

$$\sigma_2 = \beta_2 \pi \ell_{\rm pl}{}^2 (\ell_{\rm pl}/\lambda)^b = \beta_2 \pi E_\gamma{}^b M_{\rm pl}{}^{-(b+2)} \,, \tag{3}$$

where β_2 is a dimensionless constant. Given the present uncertainties, the index b is left as a free parameter; however, a simple phase-space argument suggests a lowest order value of $b = 5$.

With the number density and cross section specified, the optical depth for two particle down-scattering takes the form

$$\tau_2 = \alpha \beta_2 \pi (d_*/\ell_{\rm pl})(E_\gamma/M_{\rm pl})^b \,. \tag{4}$$

This result can be expressed in terms of the path length required for the optical depth to exceed unity. As photons travel across the universe, they will experience a quantum foam cutoff at a distance scale $d_* = (\ell_{\rm pl}/\pi)(M_{\rm pl}/E_\gamma)^b$. For example, if we take $\alpha = 1 = \beta_2$, $b = 5$, and $M_{\rm pl} \approx 10^{19}$ GeV, this quantum foam cutoff becomes

$$d_* \leq 450 \,{\rm Mpc} \,(E_\gamma/10^7 {\rm GeV})^{-5} \,. \tag{5}$$

For path lengths that are comparable to the cosmological horizon scale, one must take into account the expansion of the universe and the redshifting of photons with cosmological time. With this generalization, the optical depth takes the form

$$\tau_2 = \frac{\alpha \beta_2 \pi c}{\ell_{\rm pl} H_0} \left(\frac{E_\gamma}{M_{\rm pl}}\right)^b \int_a^1 \frac{a^{1/2} da}{a^b \left[\Omega_M + \Omega_V a^3\right]^{1/2}} \,, \tag{6}$$

where H_0 is the Hubble constant and a is the cosmic scale factor. We have assumed a spatially flat universe with matter density $\Omega_M = 0.3$ and constant vacuum energy density $\Omega_V = 0.7$, in concordance with current observations [8].

The Planck mass can be lower than its standard value $M_{\rm pl} \approx 10^{19}$ GeV. Many recent papers [9] explore the possibility of a smaller scale for quantum gravity (lower Planck mass) and larger extra dimensions in string theory. While these theories have 10 or 11 space-time dimensions, the calculation of space-time foam continues to predict one virtual black hole per Planck volume [10]. The quantum foam cutoff constructed in this letter can be used to constrain the value of the Planck mass in this context. The cross section depends sensitively on the Planck scale (eq. [3]) so that interactions of photons with virtual black holes become far more likely with a lower Planck mass.

Figure 1 shows the maximum propagation distance as a function of photon energy E_γ using the interaction cross section with $b=5$ and varying values of the quantum gravity scale $M_{\rm pl}$. Because high energy photons have already been observed from astronomical sources [11,12], a portion of the plane is already known to be unaffected by quantum foam; this region is shown as the shaded part of the plane (see also Refs. [13,6]). In addition to possible interactions with virtual black holes, high energy gamma rays can interact with photons from the radiation backgrounds of the universe. Gamma rays with energies $E > 300$ TeV can scatter off photons from the cosmic microwave background radiation (CMB) and produce e^+e^- pairs. The mean free path for pair production is only 10 kpc for photons above the energy threshold [14]; this bound is shown as the dashed horizontal line in Figure 1. For the standard value of the Planck mass ($M_{\rm pl} \approx 10^{19}$ GeV), the bound from quantum foam becomes more restrictive (with a path length less than 10 kpc) than that due to the CMB for photon energies $E_\gamma > 10^8$ GeV. Another bound arises from interactions with the cosmic background of infrared photons. The number density of photons in the infrared background is smaller than that of the CMB, and the energy threshold is lower (30 TeV). For photon energies $E_\gamma > 30$ TeV, the mean free path is about 1 Mpc [14]; this bound is shown as the dotted horizontal line in Figure 1. For the standard value of the Planck mass, the bound from quantum foam is more restrictive than that from the infrared background for $E_\gamma > 3 \times 10^7$ GeV.

These quantum foam bounds become stronger for lower values of the Planck mass $M_{\rm pl}$. Figure 1 shows that existing data rule out quantum gravity scales lower than $M_{\rm pl} \sim 10^{15}$ GeV for cross sections with $b = 5$. A more general bound can be obtained from the entire $b - M_{\rm pl}$ plane. Observed

high energy photons from extragalactic sources (with given energy E_γ and known distance d_*) must have $\tau_2 < 1$ and imply a limit on the Planck mass as a function of the index b. The observations that place the tightest limits are those with the highest energies and largest path lengths. For example, $10 - 20$ TeV photons have been detected from Mkn 421 and Mkn 501 [11] at redshifts of $z = 0.031$ and 0.033 ($d_* \approx 140$ Mpc). Many other sources have been observed with photon energies $E_\gamma = 0.3 - 10$ TeV and distances $d_* = 100 - 500$ Mpc [12]. In Figure 2, this set of observations is depicted as the dark band in the $b - M_{\rm pl}$ plane. The region below the band is ruled out, whereas the region above the band remains viable.

The bounds discussed in this paper require four conditions: (1) The vacuum is described by the paradigm of space-time foam, where virtual black holes flicker in and out of existence with a mean density of one virtual black hole per Planck volume. (2) The virtual black holes driving this effect do not preserve the identity of the photons they absorb. (3) The cross section for absorption followed by two particle emission has the assumed form (eq. [3]) in the long wavelength limit, which requires that (4) Lorentz invariance is violated (or modified [6,7]) at the Planck scale.

Existing astronomical observations already constrain theories of quantum gravity. The results shown in Figures 1 and 2 indicate that quantum gravity is constrained by at least one of the following conditions: (A) The Planck mass must be relatively large ($M_{\rm pl} > 10^{15}$ GeV), or (B) The interaction cross sections must be highly suppressed over their expected values (either $b \gg 5$ or $\beta_2 \ll 1$), or (C) Quantum gravity does not violate (or modify) standard Lorentz invariance. Future astronomical observations will probe more of the parameter space for which quantum foam can affect photon propagation and will thereby provide even tighter limits. Observations of high energy sources ($E_\gamma > 20$ TeV) out to greater distances ($d > 200$ Mpc) will provide the first new constraints. This type of observation will be limited when photon energies approach the threshold at 300 TeV due to interactions with CMB photons. To make further progress, extremely high energies ($E_\gamma > 10^8$ GeV) are needed.

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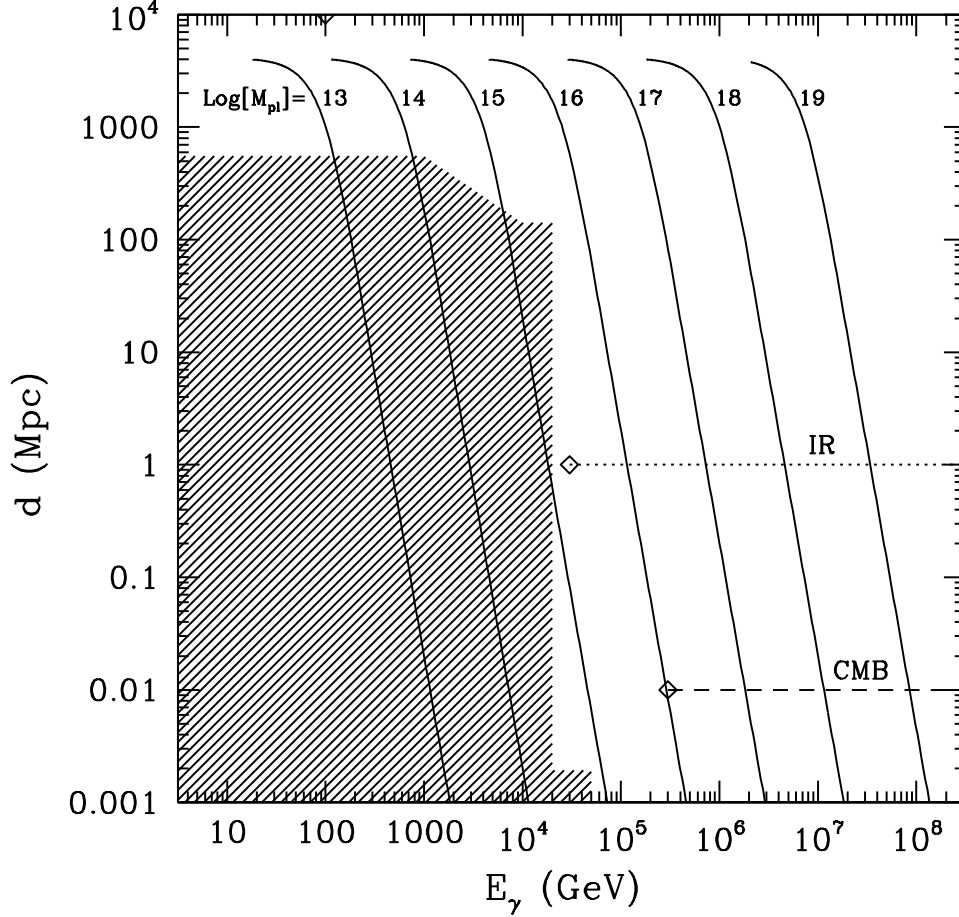[14] P.J.E. Peebles, *Principles of Physical Cosmology* (Princeton Univ. Press, Princeton, 1993) p. 182.

Fig. 1.— The predicted quantum foam cutoff as a function of photon energy for varying values of the Planck mass. The maximum distance d_* for which astrophysical photons can propagate is shown as a function of observed photon energy E_γ (i.e., present-day energy). The cross section is assumed to have the form given by equation (3) with $b = 5$. The Planck mass varies from 10^{13} to 10^{19} GeV, as indicated near the top of each curve. The shaded region shows the portion of the plane that has been probed by astronomical observations of high energy photons (see text). The dashed curve labeled CMB shows the maximum path length due to scattering of high energy photons by the cosmic microwave background; this cutoff at 10 kpc operates for photon energies $E_\gamma > 3 \times 10^5$ GeV. The dotted curve labeled IR shows the maximum path length due to the infrared background; this cutoff at 1 Mpc operates for photon energies $E_\gamma > 3 \times 10^4$ GeV.
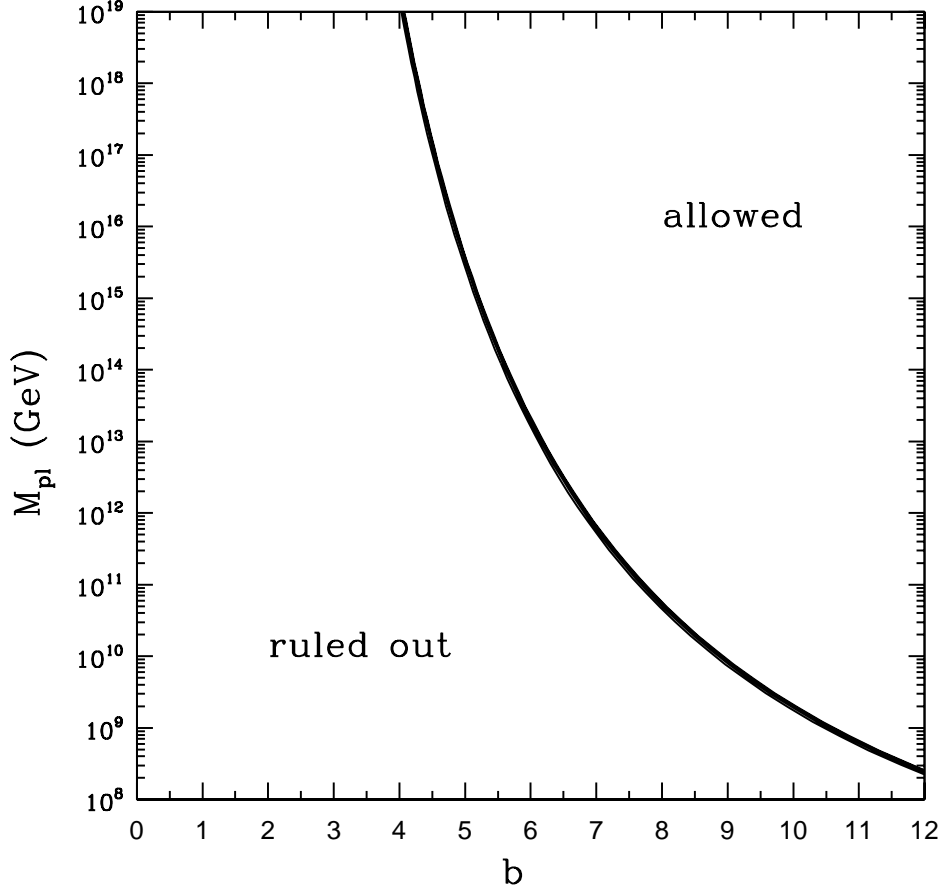
Fig. 2.— Constraints on the $b - M_{\mathrm{pl}}$ plane from the quantum foam cutoff. The horizontal axis corresponds to the index b that appears in the interaction cross section; the vertical axis shows the Planck mass M_{pl} (which can be lower than the standard value). Existing observations of high energy photons from extragalactic sources constrain the possible interactions between photons and virtual black holes. The dark band depicts the region of the plane probed by observations with photon energies $E_\gamma = 20$ TeV and source distances $d = 100 - 500$ Mpc. The allowed region of the plane is above the curves on the upper right; the region to the lower left of the curves is ruled out.